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Brane constructions, fractional branes and anti-de Sitter domain walls

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Abstract: Compactifications of type IIB string theory on $AdS_5 \otimes X_5$, where X_5 is an Einstein space, can have one-fourth or half maximal supersymmetry for certain choices of X_5. Some of these theories admit exotic domain walls arising from 5-branes wrapping 2-cycles in X_5. We explore the relationship among these domain walls, fractional branes and branes stretched on intervals. World-volume fluxes in the wrapped branes play an important role in the analysis. We draw some parallels between the AdS background with exotic domain walls and $\mathcal{N} = 1$ supersymmetric MQCD, and identify other extended objects on the AdS side in the dual brane construction. The process of brane creation is used to give an alternate derivation of the relationship between fractional branes and branes on intervals.

Keywords: M-Theory, Brane Dynamics in Gauge Theories, Supersymmetry and Duality, Supergravity Models.

Contents

1. Introduction

The study of branes at singularities has provided remarkable new insight into geometry and gauge theory. Within this circle of ideas, one should include the study of field theories on branes, both at quotient singularities [1, 2] and at non-quotient singularities such as conifolds [3], and the large-N limits of these two classes of configurations which give rise to string and M-theory compactifications on $AdS_p \otimes X_q$ [4, 5, 6]. Recently, useful knowledge about these models has been obtained using brane constructions, following the original idea of Hanany and Witten to realise field theories by suspending branes between branes.

The simplest example of the above class of models is to consider a smooth space transverse to the brane, (a trivial singularity, so to speak). For D3-branes in type IIB string theory, the transverse space is $\mathbb{R}^6$ and the large-N limit is believed to be dual to the string theory on $AdS_5 \otimes S^5$. This arises because $\mathbb{R}^6$ is a half-line fibred over an S^5 whose size varies with distance from the origin, and the large-N limit "blows up" the varying S^5 to instead have a constant size. This leads naturally to the expectation that the large-N limit will have a nontrivial effect on any singularity that one may introduce.

Continuing to work with D3-branes in type IIB string theory, the simplest singularity is a $\mathbb{Z}_n$ ALE space. Here the transverse space to the branes is $\mathbb{R}^2 \otimes (\mathbb{R}^4/\mathbb{Z}_n)$ where $\mathbb{Z}_n$ has the obvious action as a subgroup of SU(2) on the complex coordinates of $\mathbb{R}^4$. In this case, there is an entire fixed plane of singularities transverse to the branes. In the large-N limit, this singular locus gets reduced to the intersection of the fixed plane and an S^5, which is a fixed circle. The spacetime description becomes $AdS_5 \otimes (S^5/\mathbb{Z}_n)$. Half the supersymmetries are broken in this case, as compared with the previous case, thus the D3-brane gauge theory has $N = 2$ supersymmetry.

This example can be extended to $\mathcal{N} = 1$ supersymmetry in two distinct ways. One is to quotient $\mathbb{R}^6$ by the action of a group Γ that sits naturally in SU(3), for example $\mathbb{Z}_3$ or $\mathbb{Z}_k \otimes \mathbb{Z}_{k'}$. For the $\mathbb{Z}_3$ case, the transverse space to the branes initially has a point singularity at the origin, hence the large-N limit gives rise to compactification on a completely smooth space. In the latter case, there are typically intersecting singularities both before and after taking the large-N limit, though in special cases the space may be smooth.

The other way to extend these examples is to consider D3-branes transverse to more general singularities that are not quotient singularities. The prototype of these examples is the conifold singularity of a Calabi-Yau manifold. In this case one again gets $\mathcal{N} = 1$ supersymmetry on the brane world-volume. The transverse space has a point singularity, the node of the conifold, while in the large-N limit one has string theory on $AdS_5 \otimes T_{1,1}$ where $T_{1,1}$ is a smooth Einstein space [7] which is the "base" of the conifold [8]. The CFT on the D3-brane at a conifold was analysed in ref. [3] and, via brane constructions, in refs. [9, 10].

There are more general 6-dimensional singularities than the conifold (the A-D-E generalized conifolds were first described in ref. [11] in the context of noncritical strings, and subsequently studied in the brane context in refs. [12, 13]), which tend to have extended singular loci rather than a single node. The large-N limits of these cases have also been investigated [14, 15, 9, 16, 17, 18].

There are other quotients of the basic configuration which correspond to orientifolding. Though we will not discuss them here, they too, present many interesting features.

In all the examples described above, one can give a dual "brane construction" by performing a suitable T-duality. For example, the theory of N D3-branes at a $\mathbb{Z}_n$ ALE space is T-dual to a configuration of n parallel NS 5-branes in type IIA string theory, with N D4-branes stretching between them [19, 20]. This is the so-called $\mathcal{N} = 2$ "elliptic" model [21]. For more general quotient singularities one gets "brane box" configurations [22, 23]. In these models the spectrum and the superpotential can be read out using definite rules given in ref. [22]. (However, as was shown recently in an interesting paper by Aganagic et. al. [24], these rules, according to which the superpotential is obtained by drawing closed triangles across the boxes, are not true in general. To get the right quartic superpotential one has to use the "diamond" rule [24].[1] These rules reduce to the triangle rules only in special circumstances.) Similarly, the theory of N D3-branes at a conifold is dual to a configuration of perpendicular NS 5-branes with N D4-branes stretching between

[1]This resolves a puzzle that was noted in ref. [10].

them [9, 10]. Hence it is possible to look for direct correspondences between brane constructions and AdS compactifications.

Since we will be interested in relating the above ideas to domain walls, let us briefly review the relevant known results. For concreteness, consider first the system of N D3-branes at a conifold. In the $AdS_5 \otimes T_{1,1}$ model, it was argued [25] that there are two kinds of domain walls in the AdS_5 spacetime. A domain wall in 5-dimensional spacetime must be some kind of 3-brane. In the present case one can introduce the D3-branes of type IIB string theory, or one can take a D5-brane and wrap it on a 2-cycle of the Einstein space $T_{1,1}$. We will refer to the latter kind of domain wall as "exotic". (Alternatively one can also wrap an NS 5-brane, which is dual to the D5-brane under S-duality.) Domain walls can be oriented in different ways within AdS, we will consider the orientations which preserve all the supersymmetry (thus the brane is parallel to the AdS_5 boundary).

The key property of domain walls in AdS compactifications is that the radius of the AdS space jumps when one crosses them. In terms of the dual gauge theory, the rank of the gauge group increases or decreases across a domain wall. For the maximally supersymmetric $AdS_5 \otimes S^5$ compactification, the gauge group goes from $\mathrm{SU}(N)$ to $\mathrm{SU}(N+1)$ across a domain wall made up of a D3-brane. This is the only possible domain wall in this case, since S^5 has no 2-cycles.

In the $AdS_5 \otimes T_{1,1}$ case, the gauge group of the CFT is $\mathrm{SU}(N) \otimes \mathrm{SU}(N)$. Inserting a D3-brane changes the gauge group to $\mathrm{SU}(N+1) \otimes \mathrm{SU}(N+1)$. However, the exotic domain wall obtained by wrapping a D5-brane on a 2-cycle of $T_{1,1}$ ($T_{1,1}$ is topologically the same as $S^2 \times S^3$) changes the gauge group to $\mathrm{SU}(N+1) \otimes \mathrm{SU}(N)$.

The standard domain wall is easy to understand in the brane construction: its effect is to add an extra D4-brane in the elliptic models, wrapping all the way around the compact direction. As we will argue in some detail, the exotic domain wall corresponds to a D4-brane stretching part of the way around a compact direction and then ending on parallel NS 5-branes (this has been independently noted in ref. [16]).

That such an object can correspond to a wrapped D5-brane in the T-dual picture is initially surprising, since the T-duality direction would appear to convert the D4 to a D3-brane. However, this discrepancy of 2 dimensions is resolved by the observation that such branes in a brane construction are actually "fractional branes" [26] of the T-dual theory. Fractional branes are interpreted as wrapped branes of 2 higher dimensions, wrapping a 2-cycle of vanishing size that is hidden in the singularity. They are supposed to acquire their charge and tension from a flux of the 2-form field $B_{NS,NS}$ through the vanishing 2-cycle, though we will see below that this is not the complete explanation.

Since the large-N limit for N D3-branes at a conifold effectively blows up a vanishing 2-cycle of the conifold to a finite S^2, the fractional branes at a conifold blow up into exotic domain walls. Since fractional branes have already been associated in refs. [20] with branes stretched on an interval, the relation between the latter objects and domain walls seems quite natural. However, we will see that world-volume field strengths on the wrapped branes play a crucial role in providing the right properties that are predicted by various dualities.

We will also provide an independent argument for the relationship between domain walls and stretched branes using the process of "brane creation" [27].

This paper is organized as follows. In section 2 we review the properties of 3-branes at a $\mathbb{Z}_2$ ALE space and explain how fractional branes arise and how they are described in a T-dual picture. Although much of this material is known, we clarify the role of world-volume fluxes and the relationship of this system to the process of tachyon condensation via a vortex solution on a brane anti-brane pair, which has been extensively discussed of late [28, 29, 30]. The tachyon is actually absent in the limit relevant to our problem, but the field strength associated to the would-be vortex causes the pair to annihilate into a lower brane. In section 3 we extend these considerations to the conifold singularity. In particular, we observe that the Kähler transition at $B_{NS,NS} = 0$, T-dual to the crossing of two NS 5-branes, corresponds to a jump in the total world-volume field strength. In section 4 we turn to the large-N limit and the associated exotic domain walls. These are identified with a T-dual description of branes on an interval. We provide a more complete picture of the jump in the gauge group across such domain walls in the light of our observations about world-volume fluxes. We also relate the model in the presence of such domain walls to MQCD. In section 5 we examine other wrapped branes in the $AdS_5 \otimes T_{1,1}$ model, and describe the corresponding objects in the T-dual brane construction. Finally, in section 6 we provide an alternate derivation of the relationship between domain walls and branes on intervals using the phenomenon of brane creation.

2. D4-branes on an interval: the $\mathcal{N} = 2$ case

Consider a pair of NS 5-branes in type IIA theory. Let us start by taking them parallel, filling the directions $(x^1, \ldots, x^5)$, coincident in (x^7, x^8, x^9) and separated by a finite interval along x^6. Take the x^6 direction to be compact. Now take a D4-brane that terminates on each of the NS 5-branes as a 3-brane along (x^1, x^2, x^3), and stretches between them along x^6 (fig. 1). Next we T-dualize this circle.

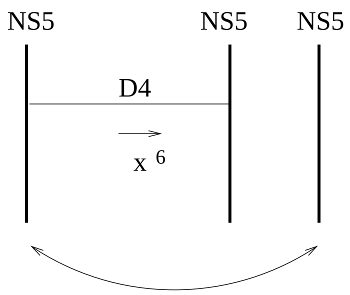

Figure 1: D4-brane on an interval.

The result of this T-duality was considered in ref. [20] which used the fact that NS 5-branes turn into Kaluza-Klein monopoles of type IIB theory. Coincident KK monopoles describe the near-horizon behaviour of an ALE singularity. As they are T-dual to type IIA NS 5-branes, they carry tensor multiplets on their worldvolumes. In [20] it was proposed that the D4-brane on an interval turns into a fractional brane [26] at the ALE singularity.

To see this, let us start with a singular $\mathbb{Z}_2$ ALE space[2] along directions x^6, x^7, x^8, x^9. The node is really a 5-plane filling the remaining directions. Close to the singular point, the space can be replaced by a 2-centre Taub-NUT metric with coincident centres. This is equivalent to saying that we have two coincident Kaluza-Klein monopoles. We also know [31] that the $\mathbb{Z}_2$ orbifold hides half a unit of $B_{NS,NS}$ flux through the shrunk 2-cycle Σ. The four moduli associated to this ALE space are three geometrical parameters, which can be thought of as the blowup of the ALE to form a smooth Eguchi-Hanson metric, and the $B_{NS,NS}$ flux.

Take a D3-brane transverse to the ALE space, filling the directions x^1, x^2, x^3. (More generally we start with N such D3-branes.) When the ALE space is singular, the world-volume theory of the 3-brane has two branches: a Higgs branch, when the brane is separated from the singularity along x^6, x^7, x^8, x^9, and a Coulomb branch when the brane hits the singularity and dissociates into a pair of "fractional branes" which can move around only in the x^4, x^5 directions. However, if the ALE space is blown up, then the Coulomb branch is lifted.

The fractional branes are interpreted as a pair of D5-branes whose 5-brane charges cancel (hence they are really a D5-brane — anti-D5-brane pair). However, they carry 3-brane charge by virtue of the CS coupling on D5-branes. Denoting the world-volume gauge field strength on the D5-brane by F_1, we have the coupling

$$\int (B_{NS,NS} - F_1) \wedge D^+ \,, \tag{2.1}$$

where D^+ is the self-dual 4-form potential in the type IIB string. At the orbifold point we have $\int_\Sigma B_{NS,NS} = 1/2$ and hence half a unit of D3-brane charge. There is an apparent puzzle here, since the anti-5-brane (whose world-volume gauge field strength is F_2) will have a coupling

$$-\int (B_{NS,NS} - F_2) \wedge D^+ \,, \tag{2.2}$$

Hence we would expect the anti-5-brane to acquire a $-1/2$ unit of D3-brane charge in this way, resulting in a net D3-brane charge of 0, while we require it to be 1. The resolution of this puzzle comes from the fact that the B-field flux is not gauge-invariant. By gauge transformations, it can be effectively made into a periodic variable taking values from 0 to 1. The correct gauge-invariant quantity on a brane is $B_{NS,NS} - F$ which appears in the couplings above.

Hence what really happens is the following. Viewed as a spacetime field, $B_{NS,NS}$ has a flux of 1/2 (which is the same as $-1/2$ since it is a periodic variable). From

[2]The considerations in this section have a straightforward extension to the case of $\mathbb{Z}_n$, although extending them to the other discrete groups of D_n or E_n type could be somewhat nontrivial.

eqs. (2.1) and (2.2), this contributes 1/2 a unit of D3-brane charge to the wrapped 5-brane and $-1/2$ to the anti-5-brane. Now let us also turn on a world-volume gauge field strength F_2 on the anti-brane and give it a flux of +1 unit through the vanishing 2-cycle Σ (more generally, we assign unit flux to the relative gauge field $F_- = F_2 - F_1$). In this configuration, the 5-brane anti-5-brane pair has total 3-brane charge equal to 1.

Examining eqs. (2.1) and (2.2) above, we see that variations of the B-field act equally and oppositely on the brane and anti-brane. This changes the relative 3-brane charge on each of them in such a way that the sum remains constant. Thus it is not quite correct to say that the total 3-brane charge and tension of the brane anti-brane pair arise from the B-flux. Such a statement is valid only if restricted to an individual brane, where one can always gauge away the world-volume field strength by a gauge transformation on $B_{NS,NS}$.

Next, perform a T-duality along x^6. Then the pair of Kaluza-Klein monopoles turns into a pair of NS 5-branes in type IIA string theory. We expect that the B-field turns into a geometrical modulus. In fact, it has been argued [20] that it becomes the separation of the NS 5-branes along the x^6 direction. The way to see this is that a D3-brane at a $\mathbb{Z}_2$ ALE space has the following terms [1] in its world-volume action[3] which depend on the fluxes of the 2-form fields $B_{NS,NS}$ and B_{RR} through the vanishing cycle of the ALE space:

$$\int \frac{1}{2}(b_1\, F \wedge *F + b_2\, F \wedge F)\,, \tag{2.3}$$

where

$$b_1 \sim \int_\Sigma B_{NS,NS}\,, \qquad b_2 \sim \int_\Sigma B_{RR}\,. \tag{2.4}$$

The above formula is actually valid for one fractional 3-brane (say the wrapped D5-brane) and we should replace $B_{NS,NS}$ by $B_{NS,NS} - F_1$. An analogous formula holds for the other fractional 3-brane (the wrapped anti-D5-brane) with $B_{NS,NS} - F_1$ replaced by $-(B_{NS,NS} - F_2)$. It follows that, with $\int_\Sigma F_1 = 0$ and $\int_\Sigma F_2 = 1$, the gauge couplings of the two U(1) gauge groups are given by $\int_\Sigma B_{NS,NS}$ and $(1 - \int_\Sigma B_{NS,NS})$ respectively. Hence T-dualizing along this direction produces a pair of NS 5-branes in type IIA theory whose separation along the x^6 circle is proportional to $\int_\Sigma B_{NS,NS}$ (in the other direction the separation is therefore proportional to $(1 - \int_\Sigma B_{NS,NS})$).

The mapping between parameters has been discussed in ref. [20]. We will review and extend this analysis in the light of our observations about the role of world-volume gauge field fluxes on the brane. In the brane construction, the two

[3] It is known [32] that couplings of this type on D3-branes receive instanton corrections which convert them into modular forms under $SL(2,\mathbb{Z})$ S-duality (a recent discussion can be found in ref. [33]). It would be interesting to understand the analogous corrections in the present case.

NS 5-branes can move around on the x^6 circle or they can separate from each other along x^7, x^8, x^9. These four possible motions must correspond, on the orbifold side, to the four deformation parameters associated to the orbifold string theory: three geometrical deformations (two complex and one Kähler) and a $B_{NS,NS}$-field modulus. On the Higgs branch, the D3-brane is separated from the ALE singularity along x^6, x^7, x^8, x^9. In the T-dual type IIA picture, the 4-brane is separated from the NS 5-branes along x^7, x^8, x^9 and has a Wilson line along x^6 in its world-volume. In particular, even taking $x^7 = x^8 = x^9 = 0$, so that the D4-brane touches the NS 5-branes, it cannot split into pieces stretching on intervals as long as there is a Wilson line.

Going to the Coulomb branch, by tuning all of x^6, x^7, x^8, x^9 to 0, the picture is somewhat different. At this point the D3-brane splits into a pair of fractional branes which can move independently along x^4, x^5. The geometric orbifold singularity now cannot be blown up any more. This is easy to see on the T-dual type IIA side, where the D4-brane splits into two pieces that stretch along the two intervals between the two NS 5-branes (one from each side of the x^6 circle). These partially wrapped 4-branes can move independently along the NS 5-branes, namely in the x^4, x^5 directions (fig. 2).

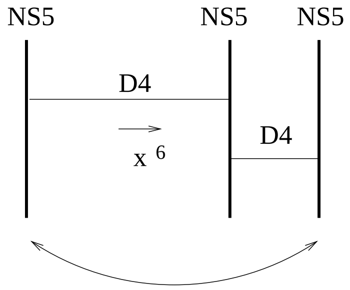

Figure 2: Wrapped D4-brane splits in two.

When the D4-brane segments are separated, the NS 5-branes cannot be separated along x^7, x^8, x^9 without some cost in energy. This is dual to the statement in the type IIB theory that once a D3-brane has reached the orbifold and split into fractional branes (i.e. on the Coulomb branch), the orbifold singularity can no longer be resolved while preserving supersymmetry.

We note in passing that since the D4-brane can split into two pieces which can separate to arbitrary distances, by cluster decomposition we would expect each separate piece (a D4-brane stretched on an interval) to be a valid state in the theory [26]. In type IIB language, this is equivalent to saying that a single fractional brane makes sense, though the process of bringing a D-brane to a Z_2 singularity always generates fractional branes in pairs.

On the Coulomb branch, the NS 5-branes are still free to move around in the x^6 direction, as they were in the Higgs branch, with the position being T-dual to the flux of $B_{NS,NS}$. Now we see that this motion also changes the relative tensions of the D4-branes stretched along intervals (we are referring to the tensions from a (3+1)-dimensional viewpoint), though the sum remains constant. These two facts are mutually consistent only because, as we have pointed out above, the B-field couples oppositely to the two fractional branes as exhibited in eqs. (2.1) and (2.2).[4]

[4]Thus in particular, we disagree with a claim made in ref. [20] (below eq. (12)) that the motion of the NS5-branes in the type IIA picture is related to Wilson lines.

To summarise, the type IIB picture on the Coulomb branch is that the relative world-volume gauge field strength F_- on the 5-brane anti-5-brane pair must be turned on over the 2-cycle Σ and gives rise to a 3-brane in the space transverse to that cycle. The spacetime $B_{NS,NS}$ flux over Σ changes the relative tensions of the wrapped brane and anti-brane keeping the total constant. In the T-dual type IIA picture, the $B_{NS,NS}$ flux corresponds to the relative location of the NS 5-branes along x^6, which has the same effect on the 3-brane tensions of the two finitely extended D4-brane segments.

This phenomenon is reminiscent of tachyon condensation, discussed in refs. [28, 29, 30]. It is interesting to compare the two. The phenomenon in refs. [28] involves a p-brane anti-p-brane pair that supports a tachyon. One then allows the tachyon to condense in a nontrivial configuration, in other words to develop a VEV which depends on only two spatial directions and resembles a vortex solution. The tachyon is charged under the relative gauge field F_- and its condensation is accompanied by an excitation of F_- with unit flux through a 2-cycle. (This 2-cycle is usually taken to be infinite 2-space or else a 2-torus.) The condensation phenomenon takes us from an unstable configuration consisting of a p-brane anti-p-brane pair, to a stable configuration of a unit-charged $(p-2)$-brane. Multi-vortices create multi-charged $(p-2)$-branes.

Our phenomenon involving fractional branes also gives rise to a unit charged $(p-2)$-brane from a p-anti-p pair (in this case $p=5$). Again, the lower brane charge is created by a flux of F_- in the world-volume theory. However, in this case the 2-cycle over which F_- acquires a flux is of zero size. Related to this is the fact that there is no tachyon in this limit.[5] Hence, going from the brane anti-brane pair to a lower-dimensional brane does not lower energy, but instead corresponds to a marginal deformation. This means, for example, that one can also go in the opposite direction: a D3-brane in the plane of the ALE space can split into a D5 anti-D5 pair, which is precisely the phenomenon we started out to discuss. Additionally, the *distribution* of 3-brane charge and tension between 5-brane and anti-5-brane can be varied by turning on the spacetime B-field, which keeps the *total* 3-brane charge fixed.

Thus, in the $\mathcal{N}=2$ supersymmetric example discussed above, of D3-branes at an ALE space, we see how a partially wrapped brane in type IIA theory, becomes a fractional brane in type IIB. Though this has already been discussed, for example,

[5] An argument for this (due to Ashoke Sen) is the following: Consider a brane anti-brane pair wrapped on a 2-torus, with a 0-brane charge of 2 units. Under T-duality, this turns into a pair of 2-branes wrapped on the dual torus, with a nontrivial SU(2) field strength through it. The fact that this is not a configuration of minimum energy in its sector corresponds to the tachyonic instability. However, as the original torus shrinks, the dual torus becomes infinite and the SU(2) field strength (since it has constant flux through the dual torus) goes to zero. Thus in the limit, there is no tachyon.

in ref. [20], we have identified more precisely the interplay between the world-volume gauge fields and the $B_{NS,NS}$ flux, which is essential to obtain a consistent picture. World-volume fluxes on the brane anti-brane pair at an orbifold singularity were used in refs. [34, 35] in configurations where the $B_{NS,NS}$ flux was fixed at the "orbifold" value of 1/2. Here, motivated by the brane construction dual, we have explained how the $B_{NS,NS}$ flux really determines only the relative tensions of the pair. Some consequences of this fact will be relevant in section 4.

Our final observation about this system has to do with a process in which the two NS5-branes connected by D4-branes pass through each other.[6] Recall the case discussed above, of a D4-brane going around x^6, which has separated into two segments (fig. 2). Let these be well-separated from each other. Now let us bring the two NS5-branes together. In this limit, one of the segments wraps the entire circle while the other shrinks to zero length (fig. 3).

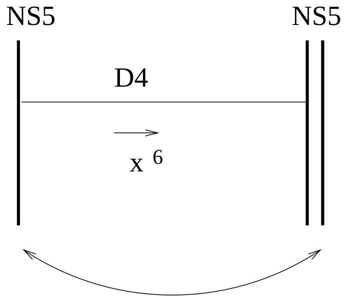

Figure 3: Two NS5-branes coinciding.

However, if we continue past this point and let the NS5-branes cross, then the first D4-brane segment wraps once around the circle and then a part of the way again. Meanwhile the other segment, which had shrunk to zero size, grows back with the *opposite* orientation (fig. 4).

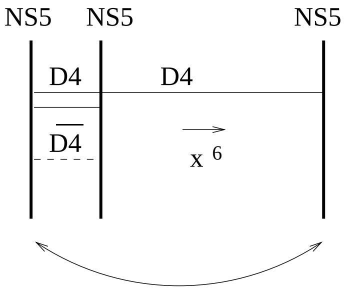

Figure 4: Two NS5-branes cross (the dotted line is an anti-D4-brane).

The result is that along one direction between the NS5-branes, there is a pair of segments of a D4-brane and an anti-D4-brane. These can annihilate, so the configuration is unstable and supersymmetry must be broken.

In the dual picture of fractional 3-branes, one of them goes to zero tension and then comes back as an anti-3-brane, while the other is like a marginally bound state of a D3-brane and a fractional 3-brane. Clearly this is a non-supersymmetric configuration.

3. D4-branes on an interval: the $\mathcal{N} = 1$ caser

Let us now generalize this to an $\mathcal{N} = 1$ example: D3-branes at a conifold singularity. This singularity is localized at a point (the origin) in the $(x^4, \cdots, x^9)$ directions. The analysis is quite similar to the previous case, except that there is really no Coulomb branch except the origin. The T-dual picture of the conifold is a pair of NS 5-branes of type IIA string theory, as for the Z_2 ALE space, except that one of the 5-branes is rotated with respect to the other [9, 10]. Thus the first NS 5-brane fills the directions

[6]This observation emerged in discussions with Ashoke Sen.

$(x^1, x^2, x^3, x^4, x^5)$ as in the ALE case, while the second one, conventionally called an NS' 5-brane, stretches along $(x^1, x^2, x^3, x^8, x^9)$. As before, they are separated along x^6 which is the T-duality direction (fig. 5).

This time the process of bringing a D3-brane (which fills the (x^1, x^2, x^3) directions) to the conifold involves tuning the 6 coordinates $(x^4, \cdots, x^9)$ to zero. In the T-dual type IIA description, the process consists of taking a D4-brane that fills (x^1, x^2, x^3, x^6), and bringing it to a fixed set of values of the remaining 5 coordinates — these are the (x^8, x^9) locations of the NS 5-brane, the (x^4, x^5) locations of the NS' 5-brane and the common x^7 location of both of them. Finally, a Wilson line in the D4-brane world-volume replaces the x^6 location of the D3-brane on the type IIB side.

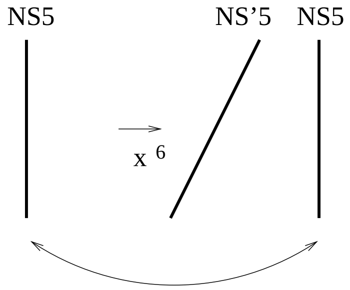

Figure 5: Brane construction dual to conifold.

Now, unlike in the $\mathcal{N} = 2$ case, the D3-brane cannot really split into fractional branes which move apart. The reason is clear in the dual picture: the rotation on one NS 5-brane has lifted the Coulomb branch. Nevertheless, it is consistent to think of the D4-brane which wraps around the x^6 direction as being made up of a pair of fractional branes which stretch between the NS 5-brane and the NS' 5-brane along opposite sides of the x^6 circle. Locally, near one of the NS 5-branes, the D4-brane will behave exactly as for the $\mathcal{N} = 2$ supersymmetric case discussed in the previous section (this fact was exploited in ref. [10] to extract the spectrum and symmetries of the $\mathcal{N} = 1$ theory). Moreover, at the level of world-volume gauge theory too, one can see the origin of the Coulomb branch: the gauge group has two SU(N) factors (assuming we brought N D3-branes to the conifold).

The main difference between this case and the ALE singularity is that here the D4-brane charge does not need to "flow" onto the NS 5-brane, but passes right through and travels around the x^6 circle. In the M-theory limit, this means that the D4 and NS 5-branes do not unify into a single M5-brane, but remain three separate components [10].

It is clear from the geometry in the type IIA T-dual picture that one can vary the gauge couplings by moving the NS 5-branes along the compact x^6 direction. Just as for the ALE case, this corresponds to changing the relative sizes of two fractional branes, even if here this is not very meaningful because the original D3-brane cannot really split into separated fractional branes. Nevertheless the phenomenon is one that we have encountered earlier, that of turning on a unit flux of F_- in the world-volume theory of a D5 anti-D5 pair, and then varying the spacetime $B_{NS,NS}$ flux. These fluxes are now through the 2-cycle of the conifold that has shrunk to zero size.

Because of the absence of a Coulomb branch, the interpretation in terms of fractional branes may appear somewhat trivial in the conifold case. However, it is possible for us to *add* a single fractional brane to this picture. On the type IIB side, we consider the state in which a single D5-brane wraps the vanishing 2-cycle of the conifold. Just as in the case of the ALE singularity, there is a special point in moduli space where this 2-cycle naturally conceals half a unit of $B_{NS,NS}$ flux, so the result is half a 3-brane. On the type IIA side, this is one extra D4-brane stretching between the NS 5-brane and NS' 5-brane along one side of the x^6 circle only (fig. 6).

Thus we see that the identification between fractional branes and branes on an interval holds for the conifold theory too.

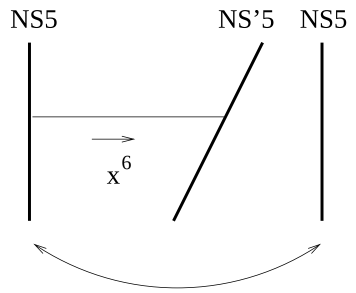

Figure 6: Fractional brane at a conifold.

This has an interesting consequence: we had noted above that in the M-theory limit, a D4-brane wrapped along x^6 becomes an M5-brane which passes through the orthogonal M5-branes at fixed locations in x^6. With the addition of a fractional brane, this no longer happens. The fractional brane will correspond, in the M-theory limit, to an M5-brane that "flares out" and joins smoothly onto the orthogonal M5-branes (fig. 7). As a result, the M-theory configuration which in the absence of a fractional brane consisted of three disconnected sets of M5-branes, becomes joined in an obvious way as soon as a single fractional brane is added to the system. The M5-branes which wrap around x^6, x^{10} are unaffected but the two M5-branes which in the type IIA limit correspond to the NS5 and NS5' branes respectively, get linked to each other. This reduces the symmetries of the (3+1)-d field theory and introduces dynamical effects into the model similar to those studied in the context of $\mathcal{N} = 1$ supersymmetric QCD in ref. [36]. We will examine some of these effects in subsequent sections.

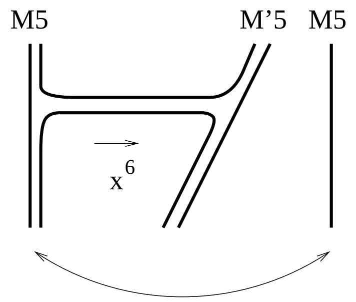

Figure 7: M-theory limit of fractional brane at a conifold.

The issues discussed above also have a bearing on the "Kähler transition" discussed in the context of the brane construction dual to generalized conifolds, in ref. [9, 18]. This is a "phase transition" in a limited sense: by separating the branes along the x^7 direction one can go around this point in moduli space, but if we keep the x^7 locations equal then we pass through a singularity. In ref. [9] it was pointed out that on the conifold (type IIB) side, this transition is a non-geometrical analogue of the well-known "flop" transition (in the latter case we would keep the 5-branes coincident along x^6 and change their x^7 separation until they pass through each other). Unlike the flop, obtained by formally varying a P^1 from positive to "negative" size, the present Kähler transition arises by varying the $B_{NS,NS}$ flux from a positive to a negative value.

From our discussion, we get some new insight into this process. We will confine ourselves to the simple conifold and ask what happens when the NS5-brane passes through the NS' 5-brane. In the absence of D4-branes stretching between the 5-branes, this transition is trivial: the configuration of an NS5-brane followed by an NS' 5-brane is equivalent to the one with reverse ordering by an overall translation along x^6. However, when a D4-brane is stretched around the x^6 circle, something more nontrivial happens. As we have seen, this D4-brane is a wrapped D5 anti-D5 pair in the type IIB description. Suppose initially $\int_\Sigma B_{NS,NS} = \epsilon$ where ϵ is a small positive number. Now vary $\int_\Sigma B_{NS,NS}$ so that it passes through zero and becomes $-\epsilon$. At this point, we need to change F_1 and F_2 so that we can re-interpret the system as having positive $B_{NS,NS}$-flux. The required change is

$$\begin{aligned} F_1 = 0 \quad &\rightarrow \quad F_1 = 1\,, \\ F_2 = 1 \quad &\rightarrow \quad F_2 = 2\,. \end{aligned} \tag{3.1}$$

This allows us to claim that the $B_{NS,NS}$ flux is $(1-\epsilon)$, which is positive and within the desired range. Notice that in the process, F_- is unchanged but F_+ has jumped. The total D3-brane number is conserved, as it must be since the system initially consisted of one D3-brane at the conifold.

Note that the jump in F_+ does not really correspond to the creation of any object as we pass through this phase transition. The world-volume action of the D5 anti-D5 pair contains the couplings:

$$\int \Big(F_- \wedge D^+ + (B_{NS,NS} - F_+) \wedge {}^*(B_{NS,NS} - F_+) + F_- \wedge {}^*F_- \Big)\,, \tag{3.2}$$

where the first term is a Chern-Simons coupling and the remaining ones come from the Dirac-Born-Infeld action.[7] The gauge transformation that we performed leaves the whole expression above unchanged.

Thus we learn that the Kähler transition for the simple conifold can be interpreted as a jump in the value of the total world-volume field strength F_+ on the brane anti-brane pair. The situation will be more interesting for the generalized A-D-E conifolds [11], considered in the present context in refs. [14, 15, 9, 16, 18], where the brane construction has several NS and NS' 5-branes and by passing them through each other the ordering is changed. This should be worth exploring in more detail.

4. Domain walls

We have been discussing two related theories, one involving N D3-branes at a Z_2 ALE singularity, and the other involving the same branes but at the node of a conifold. We will now be interested in the large-N limit of these two theories.

[7] As pointed out in ref. [37], for a brane anti-brane pair the terms which come from the DBI action, unlike the Chern-Simons terms, have to be added rather than subtracted, because they involve a scalar product rather than a wedge product of tensors.

In this limit, the former system is dual to type IIB string theory compactified on $AdS_5 \otimes (S^5/\mathbb{Z}_2)$, where $\mathbb{Z}_2$ reverses 4 of the 6 directions in which the S^5 is embedded. The latter, on the other hand, is dual to type IIB theory on $AdS_5 \otimes T_{1,1}$ where $T_{1,1}$ is the Einstein manifold discussed in ref. [7], where it is obtained as a particular quotient of the form $\mathrm{SU}(2) \otimes \mathrm{SU}(2)/\,\mathrm{U}(1)$ preserving supersymmetry. In the present context, the emergence of this space is a consequence of the fact that the conifold is a fibration of a half-line over $T_{1,1}$.

These two cases correspond to compactifications of string theory with 1/2 and 1/4-maximal supersymmetry respectively. In both cases, our goal is to understand what becomes of a fractional brane, namely a D5-brane that was wrapped around the singularity, once we introduce N D3-branes and go to large N. Since the considerations are reasonably analogous for the two cases, we will only discuss the second one in detail. For this, we will need some details of the geometry of the space $T_{1,1}$.

The metric of $T_{1,1}$ can be expressed in terms of 5 angular coordinates, ψ, θ_1, ϕ_1, θ_2, ϕ_2 as [8]

$$ds^2 = \frac{1}{6}\sum_{i=1}^{2}\left(d\theta_i^2 + sin^2\theta_i\, d\phi_i^2\right) + \frac{1}{9}\left(d\psi + cos\theta_1 d\phi_1 + cos\theta_2 d\phi_2\right)^2 , \tag{4.1}$$

where $T_{1,1}$ is topologically just $S^2 \otimes S^3$. The S^2 here is precisely the result of blowing up the vanishing 2-cycle in the conifold. The blowing up is not the usual resolution of the singularity, but rather a change in geometry induced by the distortion of space in the neighbourhood of N D3-branes.

Hence we start with the type IIB theory on a conifold, with N D3-branes located at the node. Wrap a D5-brane on the S^2 factor of the base $T_{1,1,} = S^2 \otimes S^3$. This is the domain wall of ref. [25].

As we will see below, this S^2 is really the difference (in the sense of homology) of the two S^2's which form the base of the U(1) fibration giving $T_{1,1}$. In other words, the the S^2 on which the 5-brane is wrapped to give a domain wall can be written

$$S^2 = (S^2)_1 - (S^2)_2\,. \tag{4.2}$$

In the brane construction, $(S^2)_1$ is just the compactified (45) plane while $(S^2)_2$ is the compactified (89) plane [10]. This means that the dual object to the domain wall in the type IIA picture is something that carries a charge away from the (45) plane and deposits it on the (89) plane.

A 4-brane ending on the first NS 5-brane carries away a charge from it, and if it also terminates on the NS' 5-brane then it deposits the charge on that. Moreover, since the boundary of a 4-brane ending on a 5-brane is like a 3-brane (in the (0123) directions), the transverse space to the boundary is a 2-dimensional space, namely the (45) plane or the (89) plane respectively. Hence such a 4-brane carries charge away from the (45) plane and deposits it on the (89) plane.

This shows that the dual to the domain wall is a 4-brane stretched between the NS and NS' 5-branes. It does not go all the way around the x^6 direction, but stretches along only one of the two segments between the branes. Thus, as promised, we see that the brane on an interval (which we have identified above with a fractional brane) maps onto an exotic domain wall formed by a D5-brane wrapping a cycle of *finite* size, once we go to the near-horizon AdS description.

We can exploit the relationship between this model and the $\mathcal{N} = 1$ supersymmetric MQCD model of ref. [36] to make this more precise. As we pointed out in section 3, a D4-brane stretched between an NS5 and an NS5' brane becomes joined into a single M5-brane in the M-theory limit, illustrated in fig. 7). The configuration in this figure resembles the brane construction of ref. [36] with the difference that here the x^6 direction is compact (as a result, the model with n fractional branes has gauge group $\mathrm{SU}(N + n) \otimes \mathrm{SU}(N)$ with negative β-function for the first factor and positive β-function for the second. For large N, these β-functions will only show up to subleading order in $1/N$.)

Now we are interested in going to an AdS limit, so we start with N D3-branes at a conifold singularity and then wrap a D5-brane around the S^2 cycle of $T_{1,1}$. In the brane construction, this corresponds to N D4-branes wrapping all the way around x^6 and an additional D4-brane stretching along one direction between the NS and NS' 5-branes. In the M-theory limit of this configuration, the fully wrapped D4-branes turn into N M5-branes toroidally wrapped on x^6, x^{10}. These are decoupled from the remaining branes of the problem. The single D4-brane on an interval, along with the NS and NS' 5-branes on which it ends, turns into a single M5-brane which was extensively analyzed in ref. [36]. In our conventions, it is appropriate to define $v = x^4 + ix^5$ and $w = x^8 + ix^9$ (these differ from the conventions in ref. [36] by the interchange $x^7 \leftrightarrow x^9$). We will also need the variable $t = \exp -(x^6 + ix^{10}/R)$, although this is not periodic in x^6 so we should use it only for a finite range of values of x^6. The result of ref. [36] is that the single M5-brane in question is wrapped on a holomorphic curve Σ in the complex (v, w, t) space given by (recall that there is a single D4-brane for the moment, so the parameter n in ref. [36] is equal to 1)

$$v = w^{-1} = t \tag{4.3}$$

Recalling that the v and w-planes are related [10] to what we have been calling $(S^2)_1$ and $(S^2)_2$ above, these equations give a precise meaning to our earlier statement that the fractional brane is an object wrapping $(S^2)_1 - (S^2)_2$ and stretching along x^6.

From the brane construction, it is clear that the gauge group jumps from $\mathrm{SU}(N)\otimes \mathrm{SU}(N)$ to $\mathrm{SU}(N+1)\otimes\mathrm{SU}(N)$ as we cross such a domain wall, as predicted in ref. [25] from different considerations. In our picture, the enhancement of the gauge group is due to open strings connecting the D4-brane on the interval and the remaining D4-branes to which it is parallel and coincident over a segment. However, we will see shortly that the full story is more interesting.

To complete the above arguments, we must show that S^2 of $T_{1,1}$ is $(S^2)_1 - (S^2)_2$. A basis of vielbeins on $T_{1,1}$ is given in Appendix (A) of ref. [8]. One can make various 2-forms out of these, but the question is which ones are in the cohomology. In particular, we can write the two 2-forms

$$\sin\theta_1\, d\theta_1\, d\phi_1 \pm \sin\theta_2\, d\theta_2\, d\phi_2\,, \tag{4.4}$$

both of which live only on the two S^2 factors in $T_{1,1}$, and are independent of the U(1) fibre.

Both these 2-forms can be written formally as exact forms, namely the above expressions are equal to

$$d(\cos\theta_1\, d\phi_1 \pm \cos\theta_2\, d\phi_2)\,. \tag{4.5}$$

However, the expressions in brackets above are ill-defined when any of the θ_i is equal to 0 or π, since in that case we are at the north or south pole of one of the 2-spheres and the coordinate ϕ_i is undefined there. However, the term with the + sign can be modified to

$$d(d\psi + \cos\theta_1\, d\phi_1 + \cos\theta_2\, d\phi_2) \sim de^{\psi}\,, \tag{4.6}$$

where e^{ψ} is one of the five vielbeins, and is globally defined because ψ is allowed to have gauge transformations. It follows that the term with the + sign in eq. (4.4) is genuinely exact, leaving the one with the minus sign as the representative of the second cohomology. From this in turn one deduces that in homology, the S^2 factor in $S^2 \times S^3$ is the difference of the two S^2's parametrized by θ_1, ϕ_1 and θ_2, ϕ_2 respectively, or in our brane construction, by x^4, x^5 and x^8, x^9 respectively.

Returning now to the exotic domain wall, we have observed that it is the large-N version of a fractional brane. At a Z_2 singularity or a simple conifold, a very natural configuration (the one studied in previous sections) consisted of a pair of fractional branes. The analogue in the large-N limit will be a 5-brane anti-5-brane pair wrapped around the 2-cycle of the Einstein space $T_{1,1}$.

In ref. ([25]) it was observed that while a 5-brane wrapped on S^2 is an exotic domain wall which increments the gauge group from $\mathrm{SU}(N) \otimes \mathrm{SU}(N)$ to $\mathrm{SU}(N+1) \otimes \mathrm{SU}(N)$, an anti-5-brane wrapped on the same cycle reduces the gauge group back to $\mathrm{SU}(N) \otimes \mathrm{SU}(N)$. According to our picture, the way to understand this is that the wrapped 5-brane is a fractional 3-brane, while the wrapped anti-5-brane is a fractional anti-3-brane. The two can then simply annihilate. But now from the analysis in the previous sections, we see that there is a new possibility. Suppose that on the wrapped 5-brane anti-5-brane pair we turn on a unit flux of the relative world-volume gauge field F_-. The result is that the pair annihilates into a D3-brane. On the gauge theory side, the gauge group is incremented from $\mathrm{SU}(N) \otimes \mathrm{SU}(N)$ to $\mathrm{SU}(N+1) \otimes \mathrm{SU}(N+1)$.

This can be understood in two steps. Suppose we choose to turn on a unit flux of F_- by assigning $F_1 = 0, F_2 = 1$. Then the wrapped 5-brane increases the gauge group from $\mathrm{SU}(N)\otimes\mathrm{SU}(N)$ to $\mathrm{SU}(N+1)\otimes\mathrm{SU}(N)$ while the second one further enhances it to $\mathrm{SU}(N+1)\otimes\mathrm{SU}(N+1)$. We see that the difference between the two factors of the gauge group is only partly a matter of convention. If we choose conventions where a 5-brane (with no fluxes) increments the first factor, then an anti-5-brane decreases the same factor. However, as we will show directly below, a flux of $\int F_1 = m_1$ on the 5-brane decreases both factors by m_1 units, and a flux of $\int F_2 = m_2$ increases both factors by m_2 units. As a result, a 5-brane with an arbitrary flux m_1 ends up changing the gauge group to $\mathrm{SU}(N+1-m_1)\otimes\mathrm{SU}(N-m_1)$ while an anti-5-brane with an arbitrary flux m_2 changes it to $\mathrm{SU}(N-1+m_2)\otimes\mathrm{SU}(N+m_2)$. In particular, it follows that an anti-5-brane with one unit of flux increments the second factor and not the first.

To show that world-volume fluxes have the claimed effect on the gauge group of the theory, we use an observation in ref. [25]. The baryon-like operators constructed there have quantum numbers $(N+1,1)$ and $(1,N+1)$ under the global $\mathrm{SU}(2)\otimes\mathrm{SU}(2)$ symmetry group of the theory. This can be traced to the $(N+1)$-fold degeneracy of the ground state for wrapped D3-branes (which classically are just non-relativistic charged particles localized on S^2). This degeneracy is attributed to the fact that 5-form flux through $T_{1,1}$ gives rise to magnetic flux through S^2, which imposes a minimum angular momentum on the charged particle propagating on S^2. The world-volume fluxes in our picture are simply additional to the magnetic flux coming from wrapped 5-branes. On a 5-brane, m_1 units of this flux decrease the degeneracy[8] from $N+1$ to $N-m_1+1$, implying that the baryons have SU(2) quantum numbers equal to $(N-m_1+1,1)$ and $(1,N-m_1+1)$. It follows that the world-volume flux alone would convert the gauge group to $\mathrm{SU}(N-m_1)\otimes\mathrm{SU}(N-m_1)$, which is what we wanted to show.

One may ask whether our picture of the relationship between world-volume fluxes and the tachyon condensation phenomenon on the brane anti-brane pair survives the large-N limit. The string theory on $AdS_5\otimes T_{1,1}$ is the large-N limit of N D3-branes at a conifold, and we have argued in a previous section that for any N there is no tachyon on the pair. However, in the large-N limit the geometry of the spacetime is completely modified. One consequence is that the exotic domain wall has a finite tension even in the absence of $B_{NS,NS}$-fluxes and world-volume field strengths. This tension will be of order the product of the D5-brane tension and the volume of S^2, or roughly

$$\frac{1}{g_s(\alpha')^3}\times R^2 \sim \frac{N}{\sqrt{g_sN}(\alpha')^2}\,. \qquad (4.7)$$

[8]The degeneracy is decreased and not increased because the world-volume gauge field couples with the opposite sign from $B_{NS,NS}$.

This contrasts with the fact that the corresponding object in flat space would be tensionless. Since that fact was crucial in arguing for the absence of tachyons on the brane anti-brane pair, it remains to be understood what is the correct statement that one can make about tachyons on a domain-wall anti-domain-wall pair.

The fact that the $B_{NS,NS}$ flux through S^2 changes the tension of the domain wall can be seen quite directly. In this situation there is a nonzero field strength of $H_{RR} = dB_{RR}$ through the S^3 factor on one side of the domain wall. The $B_{NS,NS}$ flux creates a net 3-brane charge, which is a flux of dD^+. Thus we have nonzero and correlated values of $B_{NS,NS}$, dB_{RR} and dD^+, as expected from the fact that the spacetime type IIB theory has a coupling $\int B_{NS,NS} \wedge dB_{RR} \wedge dD^+$.

5. Other extended objects

In the type IIB compactification on $AdS_5 \otimes T_{1,1}$ there are stable objects arising from wrapped branes over various cycles. For example, a "fat string" arises [25] from wrapping a D3-brane over the S^2 factor in $T_{1,1}$. This object can be easily identified in the T-dual brane construction. It is a 2-brane stretching between the NS and NS' 5-branes. Thus it extends for a finite distance along x^6 and for an infinite distance along one of x^1, x^2, x^3 depending on how we choose to orient the string.

In the brane construction it is clear that if we stretch a D2-brane and a D4-brane between an NS and NS' 5-brane then all supersymmetries will be broken, in agreement with the fact that this fat string is not expected to be a BPS object. However, from the fact that S^2 is a nontrivial homology cycle, the fat string appears to be stable. It is interesting to compare this string with the QCD string arising in the model of ref. [36]. Evidently the fat string is rather different. The QCD string, while also non-BPS, can annihilate in groups of n where the MQCD gauge group is SU(n). For $n = 1$, the case of a single fractional brane, the QCD string would therefore be unstable. Moreover, even for arbitrary n, it is given by an M-theory membrane wrapped in a way quite different from what we have described above. For example, while our membrane stretches along x^6, the QCD string arises from a membrane at a fixed value of x^6.

Besides the D3-brane and the D5-brane, one can also wrap NS5-branes or more generally (p, q) 5-branes on S^2. These are all obtained from a D5-brane using S-duality. On the brane construction side, the picture is clearest after going to the M-theory limit. In this limit, we have a configuration as in fig. 7. If we had started with a D5-brane, the horizontal "tube" of M5-brane in the figure would have been wrapped on the x^{10} direction while extending along the x^6 direction. If instead we start with a (p, q) 5-brane, then this tube is wrapped on a (p, q) cycle of the 2-torus whose coordinates are x^6, x^{10}.

Next, consider the D3-brane wrapped on S^3. Before taking the large-N limit, this corresponds to the much-discussed BPS state which becomes massless at the conifold point in the moduli space of a Calabi-Yau manifold [38]. At large-N, this state is no longer massless, and instead corresponds to the "baryon" of ref. [25]. After T-dualizing, this state should be a 2-brane wrapped on some combination of the x^4, x^5 and x^8, x^9 directions. This is because, as shown in ref. [10], the direction ψ in the metric of $T_{1,1}$ (see eq.4.1) is identified with the T-duality direction x^6. The cycle S^3 can be viewed as a fibration of ψ over a 2-sphere parametrized by some combination of $\theta_1, \phi_1, \theta_2, \phi_2$. According to the analysis of ref. [10], these map to the x^4, x^5, x^8, x^9 directions. Thus, T-dualizing along the x^6 direction converts the wrapped 3-brane to a 2-brane wrapped over a 2-cycle in these 4 directions.

In fact, as pointed out in ref. [10], the x^4, x^5 and x^8, x^9 pairs of directions should each correspond to 2-spheres rather than planes as they do in the brane construction. This limitation of the brane construction makes it somewhat difficult to understand in more detail the state obtained by wrapping a 2-brane over the 2-cycle above.

Finally, a D5-brane wrapping S^3 gives rise to a "fat membrane". In the T-dual picture, this object will be similar to the baryon above, except that it has two additional dimensions filling a plane in the x^1, x^2, x^3 directions. Thus it is a D4-brane stretching along, say, x^1, x^2 and a 2-cycle in x^4, x^5, x^8, x^9. Although this object does not stretch along x^6, it is wrapped on x^{10}. Hence again it will have (p,q) duals which are wrapped on a (p,q)-cycle of the x^6, x^{10} 2-torus. These will correspond in the type IIB theory to (p,q) fat membranes obtained by wrapping (p,q) 5-branes over S^3.

It is intriguing that MQCD too has a membrane in it, the QCD domain wall, described as an M5-brane wrapping a supersymmetric 3-cycle [36]. The configuration above also lifts to an M5-brane wrapping a 3-cycle, but it is not expected to be a BPS object, and the wrapping directions are rather different. Nevertheless, we find it curious that our model and MQCD both have some kind of fat string and domain wall that are described respectively by an M-theory membrane and a 5-brane. This may be related to the fact that our model in the presence of exotic domain walls resembles MQCD, as noted in section 4.

6. Fractional branes and brane creation

In this section we will give a new argument for the existence of fractional branes in conifold like models. The argument is based on the technique of brane creation. Certain configurations of branes suspended between two infinitely extended branes can be thought of as being created by crossing the two extended branes. Since we are looking for the T-dual of such models, it will be simpler to T-dualize the *initial* configuration and then represent the crossing by turning on a Wilson line in the final picture. This Wilson line will give rise to configurations which will be interpreted as the T-dual of the suspended brane.

However, as we shall see, not all configurations of suspended branes lead to fractional branes. The existence of fractional branes is due to some special properties of Taub-NUT spaces which appear as a consequence of T-duality on the infinitely extended branes. We start with a configuration of a fundamental string between two D4-branes.

Consider two D4-branes along

$$\begin{array}{lcccccccccc} \text{D4:} & 0 & 1 & 2 & 3 & 4 & - & - & - & - & - \\ \text{D4':} & 0 & - & - & - & - & 5 & 6 & 7 & 8 & - \end{array} \tag{6.1}$$

where x^9 is a compact direction. Initially the two D4-branes are at a finite distance along x^9. When they cross on the circle, an F-string is created. The reason is that a D4 is a magnetic source of the 3-form potential $C_{\mu\nu\rho}$, and when another D4 crosses the first one there is a jump in the flux of the corresponding field strength $G_{\mu\nu\rho\sigma}$. Due to the Chern-Simons coupling $\int\; G \wedge A$ on the world volume of the D4-brane (A is the gauge field) this jump leads to a coupling $\int A$ on the world-line of the intersection point. This corresponds to a source term for the world-volume gauge field, which will in general break supersymmetry. To get a BPS state, according to ref. [39], we have to excite another world-volume scalar, say x^9. Therefore when one D4 crosses another, it pulls out a piece of the second D4 as a "spike". This spike is a fundamental string [27, 40, 41].

We would like to determine the T-dual of this configuration, of two D4-branes connected by a F-string. To do this we instead T-dualize the initial configuration, of the two D4 before crossing, and then turn on a Wilson line in the final picture. The T-duality is made along x^9. The D4-branes will become two D5-branes intersecting along a (wrapped) string.

The relative velocity of the D4-branes translates into a time-varying Wilson line on the circle [41]

$$\int d^2x \left(\frac{\partial x^9_{(1)}}{\partial t} - \frac{\partial x^9_{(2)}}{\partial t} \right) \quad \xrightarrow{T_9} \quad \int d^2x \left(\frac{\partial A^9_{(1)}}{\partial t} - \frac{\partial A^9_{(1)}}{\partial t} \right). \tag{6.2}$$

The RHS is a $1{+}1d$ chiral anomaly $\int \omega\, \epsilon^{ab}\, \partial_a A_b$, where ω is the gauge-transformation parameter. This anomaly is due to a chiral fermion propagating along the intersection of the two D5 branes.[9]

The term which cancels the anomaly is

$$S = \int H_{RR} \wedge A \wedge F \tag{6.3}$$

on the world volume of each D5. $H_{RR} = dB_{RR}$, the pullback of the background three form, in the absence of any source. The cancellation takes place via anomaly

[9] Since this fermion at the intersection is chiral, we cannot give it a mass by moving the two D5-branes apart. That this is the case is apparent from the world-volume directions of the two D5 branes or, in the T-dual, of the two D4-branes. This shows that brane creation can only occur if we have have branes which together fill out eight spatial directions.

inflow. We have a coupling, eq. (6.3), in $5+1d$ spacetime. Along a $1+1d$ subspace of this, chiral fermions propagate and give rise to the anomaly eq. (6.2). Now in the original picture, of two D4-branes crossing, we saw that on *each* D4-brane there is a change of flux of G. Therefore here, since D5-branes are a magnetic source of H_{RR}, we find that changing the Wilson line produces a change of flux of H_{RR}. In other words, a gauge transformation $\delta A = d\omega$ on the world-volume will vary eq. (6.3) by $-\int dH_{RR}\wedge(\omega F)$. Since $dH_{RR}\neq 0$ in the presence of a magnetic source of H_{RR} flux, we end up with

$$\delta S = -\int d^2x\ \omega\, \epsilon^{ab}\partial_a A_b\,. \tag{6.4}$$

This is the inflow which cancels the anomaly.

Thus in the T-dual picture we get 0-branes (which are chiral fermions coming from the string joining the D5-branes) on the two D5-branes. The anomaly is cancelled by inflow. To summarise, we see that in the D4-D4 system the flux change creates a spike to preserve supersymmetry, while in the D5-D5 system the flux change creates an anomaly inflow to cancel the anomaly due to chiral fermions.

Now consider another example of a D4-brane between an NS5-brane and a D6-brane. This configuration is closely related to the configurations we have been studying in the previous sections. The orientations of the branes are as follows:

NS5:	0	1	2	3	4	5	-	-	-	-
D4:	0	1	2	3	-	-	6	-	-	-
D6:	0	1	2	3	-	-	-	7	8	9

We are considering this configuration because a D4-brane gets created when we move a D6 across NS5 along x^6, which is chosen to be a compact direction of radius R. We want to determine the resulting configuration after performing a T-duality along x^6.

The idea, as before, is to T-dualize the initial configuration of an NS5 and a D6-brane. The crossing of the branes will now be reflected as an asymptotic Wilson line on the $7+1$ dimensional gauge theory.

The reason why a D4-brane is created when we move a D6 across an NS5 is as follows. On the world volume of the NS5-brane there propagates a chiral (2,0) tensor multiplet whose fields are $(B^+_{\mu\nu}, 5\phi)$. The NS5-brane and the D6-brane are magnetic sources of the spacetime 2-form potential $B_{NS,NS}$ and 1-form RR potential A respectively. The relevant coupling on the world volume of the NS5-brane is

$$\int A\wedge *d\phi = \int A\wedge dC_4\,, \tag{6.5}$$

where $dC_4 = *d\phi$ is the six dimensional dual of a world-volume scalar. From the previous arguments we see that there is a source of $\int C_4$ on the world volume of the NS5 when it is crossed by a D6-brane. By itself, such a C_4 background will break supersymmetry. To preserve supersymmetry we need to excite a scalar x^6 which will satisfy

$$\partial^2 x^6 = \delta(x)\delta(y)\,, \tag{6.6}$$

where x, y are coordinates of the world-volume. Observe that ∂^2 involves *all* the six coordinates of the NS5-brane, but $\delta(x)\delta(y)$ depends only on two of these coordinates. Therefore the spike will be translationally invariant along three directions. This implies that we have a *four-brane*. A similar conclusion can also be arrived at by doing a series of S and T-dualities to the D4-D4 system. The T-dualities are all done orthogonal to the compact direction.

From the D6-brane side we have a coupling

$$\int B_{NS,NS} \wedge *F = \int B_{NS,NS} \wedge dC_4\,. \tag{6.7}$$

This clearly leads to the same result.

Now, as before, consider T-dualizing the initial configuration. The NS5 brane will become a Taub-NUT space and the D6-brane will become a D7-brane wrapping the Taub-NUT space (which has a non-trivial metric along the (x^6, x^7, x^8, x^9) directions). The multiplet propagating on the "Taub-NUT 5-plane", i.e. along the $(x^0, x^1, x^2, x^3, x^4, x^5)$ directions, is again $(B^+_{\mu\nu}, 5\phi)$. The self-dual antisymmetric tensor $B^+_{\mu\nu}$ comes from reducing the ten-dimensional RR 4-form $D^+_{\mu\nu\rho\sigma}$ as $B^+_{\mu\nu}(x) \otimes L_2(y)$, where x, y are coordinates along the 012345 directions and the Taub-NUT space respectively, and $L_2(y)$ is the normalizable harmonic two-form on Taub-NUT space. The five scalars can be identified as follows: two of them come from the axion-dilaton of type IIB and another two arise as an L_2 reduction of the NS-NS and RR B-fields of the type IIB theory. The fifth scalar is the gravity fluctuation. Various couplings of these fields with background IIB fields can be worked out easily starting from the non-selfdual action of IIB supergravity.

The D7-brane wrapping the Taub-NUT space will give rise to a D3-brane bound to it. The charge of the D3-brane is given by the non-trivial $B_{NS,NS}$ background on the Taub-NUT. To see this, consider some of the couplings on the world volume of the D7-brane (we are neglecting constant factors in front of each terms)

$$\int *\widetilde{\phi} + \int D^+ \wedge F \wedge B_{NS,NS} + \int D^+ \wedge F \wedge F + \cdots \tag{6.8}$$

These couplings are derived from the WZ coupling $\int C \wedge e^{B-F}$, where C is the formal sum of the RR potentials. The first term $\int *\widetilde{\phi}$ gives the charge of the D7-brane.

Now the motion of the D6 will turn into a Wilson line on the D7. The point where the two branes, NS5 and D6, touch is the zero of the Wilson line. Any positive value of the Wilson line will tell us how far apart the two branes are after crossing. However, observe that there is *no* global cycle now. Far away from the centre of the Taub-NUT, the space looks like $\mathbb{R}^3 \times S^1$ but there is no non-trivial circle at the centre. Therefore, we cannot turn on a flat connection on this space. Instead, a self-dual connection can be turned on.[10] This self-dual connection satisfies the following equation on the Taub-NUT space

$$F = dA = L_2\,, \tag{6.9}$$

where L_2 is the unique normalizable harmonic two-form on the Taub-NUT space. This harmonic two form, being normalizable, goes to zero at infinity, hence we have a flat connection there. At infinity there is an S^1 and therefore the flat connection corresponds to a Wilson line.

In such a background, we can decompose the field strength F as

$$F = L_2 + F_1\,. \tag{6.10}$$

F_1 will now appear as a gauge field on the D7 (or Taub-NUT plane). Inserting eq. (6.10) in eq. (6.8) and integrating out L_2, we have the following couplings

$$\begin{aligned}
7-\text{brane}: &\quad \int *\widetilde{\phi} + \int D^+ \wedge F_1 \wedge F_1\,,\\
5-\text{brane}: &\quad \int D^+ \wedge (B_{NS,NS} - F_1)\,,\\
3-\text{brane}: &\quad \int D^+\,.
\end{aligned} \tag{6.11}$$

The term $\int D^+$ is just the D3-brane charge. This is the usual D3 which appears as an instanton on the D7. In the original picture this will correspond to a D4 stretching all the way around the compact circle x^6.

The 5-brane term is more interesting. This also gives rise to a D3-brane charge. But the charge is measured by $\int B_{NS,NS}$.[11] Therefore we have here another source of D3-brane charge — a wrapped five brane. If $\int B_{NS,NS}$ is fractional we will get a fractional D3-brane here. Since $B_{NS,NS}$ and L_2 are defined only on the Taub-NUT space, terms like $\int C_{RR} \wedge F^m \wedge B^n$ for $m + n > 4$ do not contribute to the result. This fractional brane is related to the discussions of the previous sections. We will make the identification more precise shortly.

Another point to consider is the following. In the original picture we can have a situation in which the D6-brane is fixed but now the NS5 brane moves. In the T-dual picture for this case we will have the following sources of D3-brane charge

$$\int D^+ \wedge B \wedge B + \int D^+ \wedge B \wedge F\,. \tag{6.12}$$

[10]We would like to thank Anton Kapustin and Angel Uranga for discussions on this point.

[11]This is because, as discussed in section 3, we can gauge away the world volume field strength F_1 by a B-field gauge transformation.

The first measures the T-dual of the D4 around the circle and the second measures the D4 between between two branes. To see this observe that the role of B and F gets exchanged in eq. (6.8). This is clear from the fact that the quantity which is physically observable, i.e gauge invariant, is $B - F$; and it measures the relative distance between the two branes, NS5 and D6.

Now consider the asymptotic region in the Taub-NUT space (which is being wrapped by a D7). We have a time varying Wilson line there. This is the manifestation of the motion of a D6 on the circle x^6 in the original picture. Due to the varying Wilson line we have an *apparent* anomaly, as before, in $1+1d$. To cancel this "anomaly" we need the following WZ coupling

$$S = \int G_5 \wedge A_1 \wedge F_1 \,, \tag{6.13}$$

where $G_5 = dD^+$ in the absence of any source. Under a local gauge transformation $\delta A_1 = d\omega$ we get

$$\delta S = -\int \omega F_1 \,.$$

This equation holds because $dG_5 \neq 0$ in the presence of a D3-brane. The source in question is precisely the D3-brane whose charge is measured by the non trivial $B_{NS,NS}$ background.

Therefore we see that the T-dual of a D4-brane between an NS5-brane and a D6-brane is a D3-brane bound to a D7-brane and a Taub-NUT space. The charge of the D3-brane is given by the non trivial $B_{NS,NS}$ background on the Taub-NUT.

At this point we would like to give a consistency check for the identification of the D3-brane charges made above. Consider a situation in which, in the original picture, the D6-brane crosses the NS5-brane on a circle many times. Every time it crosses the NS5, it creates a new D4-brane between the two branes. For example, when it crosses the second time there will be a new D4 *plus* the original one which was created in the first crossing. This makes the total count as one *complete* D4, i.e a D4 starting and ending on the NS5, and two D4-branes stretched between NS5 and D6. Similarly the third crossing will give a count of three complete D4-branes plus three stretched D4's, and so on. Thus after $(n+1)$ crossings we find a total of $1+2+3+\cdots+n = n(n+1)/2$ complete D4-branes.[12] For large n, this grows as $n^2/2$. In the T-dual picture, motion along x^6 will be replaced by a background $F = nL_2$. From eq. (6.8) we see that this gives a D3-brane charge of

$$\frac{n^2}{2}\int D^+ \,,$$

[12] Another way to see this is the following: Consider an infinite sequence of NS5 branes along a *non compact* x^6 direction separated by a distance R. When a D6-brane moves along x^6 crossing all the NS5-branes, we see that the number of D4-branes increases linearly.

confirming the identification that the third term in eq. (6.8) measures the T-dual of the D4 starting and ending on the NS5. In addition to that we also have, from eq. (6.8), another term which goes as

$$n \int D^+ \wedge B_{NS,NS} \, .$$

This is just the charge of the n D4-branes, which, at the nth crossing, were *between* the NS5 and D6. The number of such D4-branes obviously grows as n. Therefore the third term in eq. (6.8) does indeed measure, in the T-dual, the charge of a D4 between the NS5 and a D6.

However in the process of creating many D4-branes by crossing the NS5 and D6-branes many times we should be careful not to violate the *s-rule* [27]. That this rule is respected can be seen from the construction of multiple images. We have a situation in which there is an infinite array of NS5-branes. The various D4-branes created by crossing D6-branes now do not join the *same* NS5-branes (although for the purpose of calculating the number of complete D4-branes we have broken the D4-branes on the array of NS5-branes). This is illustrated in fig. 8.

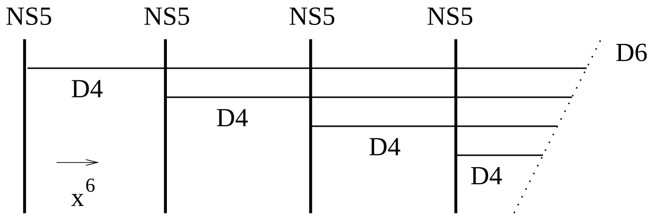

Figure 8: D6-brane crossing NS5-branes.

Now let us return to the conifold. As described in the previous sections, what we want is the T-dual of a D4-brane between two NS5-branes which are along the 12345 and 12389 directions respectively. The T-dual of two intersecting NS5-branes gives a conifold [9, 10].

Since brane creation is a local process, the above analysis tells us that the D4-brane becomes a D3 brane. We observed that in the previous case the charge of the D3 was given by the $B_{NS,NS}$ background. For the present case, the situation will be the same. On the conifold there can be a non-trivial $B_{NS,NS}$ background on the 2-cycle. Therefore the D3-brane charge will now be given by $\int_{S^2_{base}} B_{NS,NS}$. As discussed above, physically this is just a D5-brane wrapped on the base of the conifold.[13]

[13] Another way to see this is the following: For a bunch of D3-branes placed at the conifold point we know that the dynamics of the D3-branes is governed by an $\mathcal{N}=1$, SU(N)$\otimes$SU(N)$\otimes$U(1) gauge theory with matter $A_i, B_i (i = 1, 2)$ in the antisymmetric representation of the gauge group [3, 9, 10] and with a quartic superpotential. The D-term equation is given by

$$D = |A_1|^2 + |A_2|^2 - |B_1|^2 - |B_2|^2 - \zeta \, ,$$

where ζ is the coefficient of the FI term in the $3+1d$ gauge theory. This ζ is also the conifold flop factor [3]. In the T-dual picture a flop transition will be viewed as a motion of one of the NS5-brane along x^7 [42]. This motion will give mass to the D4-brane placed between the two NS5-branes.

For the case of two parallel NS5-branes the T-dual is an A_1 singularity. At the orbifold point, $\int B_{NS,NS} = 1/2$. Therefore we have a fractional D3-brane whose charge is half-integral.

Another interesting case where we can use brane creation to predict the T-dual of some configuration is the original Hanany-Witten setup. Consider the following brane configuration:

NS5:	0	1	2	3	4	5	-	-	-	-
D5:	0	1	2	-	-	-	-	7	8	9
D3:	0	1	2	-	-	-	6	-	-	-

As before we assume the x^6 direction to be compact and the D3 to lie between the two five branes. This configuration gives rise to $N = 3, d = 3$ gauge theory.

If we take a configuration of an NS5 and a D5 with *no* D3-brane between them then we can create a D3 by crossing the two branes. The analysis of brane creation for this configuration is identical to the previous discussions. The NS5-brane is a source of $B_{NS,NS}$. Therefore moving a NS5-brane across a D5 amounts to changing the $B_{NS,NS}$ flux on the world volume of the D5 brane. Due to the coupling $\int B_{NS,NS} \wedge *F$ on the D5, we see that a D3-brane gets created in the process. A similar argument can be given for the case in which a D5 crosses an NS5-brane.

Now we T-dualize the initial configuration. The NS5-brane becomes a Taub-NUT space and the D5 becomes a D6 completely wrapping the Taub-NUT space. The motion of the branes in the original picture gets replaced by an asymptotically varying Wilson line on D6 which goes from negative to a positive value.

As before, we can analyse the world volume coupling on a D6 which is wrapping a Taub-NUT space completely. Integrating out the L_2 we have the following sources of D2 brane charge

$$\int C + \int C \wedge (B_{NS,NS} - F_1) \,. \tag{6.14}$$

The first term is the usual T-dual of a D3 completely wrapping the x^6 circle. The second term will give rise to D2-brane charge from $\int B_{NSNS}$.

Asymptotically, due to a varying Wilson line we have an apparent anomaly. This is cancelled by the term $\int G_4 \wedge A_1 \wedge F_1$, $G_4 = dC$ in the absence of any source. Therefore the T-dual of a Hanany-Witten type configuration is a D2 brane bound to a D6 and a Taub-NUT space. The charge of the D2 brane is given by the background $B_{NS,NS}$ field.

We can lift this configuration to M-theory and try to see what the brane creation process implies. A configuration of an NS5-brane and a D5 brane intersecting on a

Therefore, in the conifold model, as we move the NS5 along x^7 a two cycle grows giving mass to the D3-brane. This is only possible if the D3-brane was a wrapped D5-brane. We thank Mike Douglas for discussions on this point.

$2+1$ dimensional space in type IIB theory can be described as M-theory on a toric hyper-Kähler manifold [43]. On the IIB side, as we have seen before, one can move the branes across each other to create a D3 between them. How does one interpret this in M-theory?

The IIB configuration of an NS5 and a D5-brane on a circle T-dualizes to the configuration, described above, of a Taub-NUT space and a D6-brane. When we lift that configuration to M-theory, these just become a pair of intersecting Taub-NUT spaces. This space has $Sp(2)$ holonomy and is a toric variety. The low energy dynamics of a *single* Taub-NUT space is governed by a U(1) gauge multiplet. From the above analysis we found that the T-dual of a D3-brane between two 5-branes was a D4-brane carrying a D2-brane charge. In M-theory we expect this to go to an M5-brane wrapped on a 3-cycle of the hyper-Kähler manifold, and carrying an M2-brane charge.

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